PMM U.S.S.R.,Vol.45,pp.173-178 0021-8928/82/2 0173 $7.50/0

UDC 62-50

CERTAIN IDENTIFICATION PROBLEMS FOR LINEAR DISCRETE SYSTEMS WITH QUADRATIC CONSTRAINTS*

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The matrix parameter estimation problem is examined for linear discrete systems under the assumption that the available data is limited to the measurement of a function of the system's coordinates /1—7/. A description of the data set of the weight functions compatible with the measurement results, as well as a matrix equation for its center, are obtained. It is shown that the resolving operator for the estimation problem given admits of a factorization whose properties permit the reduction of the solving of the matrix equations obtained to the solving of vector equations. In the posing of the problem and in the solution methods the paper abuts the investigations in /1,2/.

1. Statement of the problem and basic definitions. A linear controlled system with the discrete time

$$x(1) = K(1)x(0) + Bu(0) \tag{1.1}$$
$$x(k+1) = K(k+1)x(0) + \sum_{i=0}^{k-1} K(k-i)Bu(i) + Bu(k),$$
$$k = 1, \ldots, p-1$$

is given. Here $K(i)$ $(i = 1, \ldots, p)$ are hitherto unknown weight $(n \times n)$-matrices, $x(0)$ is a known n-dimensional vector of the initial position of system (1.1), $u(i)$ $(i = 0, \ldots, p-1)$ are known r-vector-valued controls. The matrix B of size $(n \times r)$ is known. Additional information on system (1.1) can be obtained during the process at the expense of additional measurements relative to the equation

$$y(k+1) = Gx(k+1) + \xi(k+1), \quad k = 0, 1, \ldots p-1 \tag{1.2}$$

where $\xi(i)$ $(i = 1, \ldots, p)$ are m-vector-valued noise in the measuring device, G is known $(m \times n)$-matrix. It is assumed that the a priori unknown parameters of system (1.1), (1.2) are subject to the consistent quadratic constraints

$$\sum_{i=1}^{p} \{\langle K(i) - K_0(i), N_*(K(i) - K_0(i))\rangle + (\xi(i), \xi(i))\} \leqslant \mu^2 \tag{1.3}$$

Here N_* is a linear symmetric positive operator on the space of $(n \times n)$-matrices (in particular, it can be operator of left or right multiplication by a symmetric positive $(n \times n)$-matrix), $K_0(i)$ $(i = 1, \ldots, p)$ are known $(n \times n)$-matrices, μ is a prescribed number. The symbol $\langle \cdot, \cdot \rangle$ henceforth denotes the scalar product on matrix spaces, defined $\langle A, B\rangle = \operatorname{tr}(AB^*)$ /8—14/, while $(\cdot, \cdot)$ denotes the scalar product on vector spaces. We introduce the following definitions and notation.

Definition 1.1. The data set $\mathbf{K}(p) = \mathbf{K}(p \mid y(1), \ldots, y(p))$ of weight functions of system (1.1)—(1.3) is the family of all those and only those sequences $\{K(1), \ldots, K(p)\}$ of matrices $K(i)$, for which m-vectors $\xi(1), \ldots, \xi(p)$ exist such that constraints (1.2)—(1.3) are fulfilled.

Everywhere below the measurements $y(1), \ldots, y(p)$ are fixed, $p \geqslant 1$ is a fixed number. Let X be a finite-dimensional Euclidean space, A be a convex compactum lying in X, the function $\varphi(x) = \max\{\|x - z\| \mid z \in A\}$.

Definition 1.2. $a \in X$ is called the center of the convex compactum A if $\varphi(a) = \min\{\varphi(x) \mid x \in A\}$.

We see that the convex compactum A always has a center $a \in A$ and a is a center of the ellipsoid $(x - a, M(x - a)) \leqslant \nu^2$ if M is a linear positive operator on X.

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*Prikl.Matem.Mekhan.,45,No.2, 241-248,1981

Problem. Determine the data set $\mathrm{K}(p)$ for system (1.1)—(1.3) for a specified initial state $x(0)$, an input $\{u(0), \ldots, u(p-1)\}$, and measurement results $\{y(1), \ldots, y(p)\}$. Find the conditions determining the center of $\mathrm{K}(p)$.

It is shown below that in the problem at hand the data set is an ellipsoid in the corresponding space of matrices. It is convenient to introduce the following notation (the asterisk denotes transposition):

$$\mathbf{u}=\{u(0),\ldots,u(p-1)\}^*\in\prod_{i=0}^{p-1}R^r,\quad \mathbf{x}=\{x(1),\ldots,x(p)\}^*\in\prod_{i=1}^{p}R^n$$

$$\mathbf{y}=\{y(1),\ldots,y(p)\}^*\in\prod_{i=1}^{p}R^m,\quad \boldsymbol{\xi}=\{\xi(1),\ldots,\xi(p)\}\in\prod_{i=1}^{p}R^m$$

$$\mathbf{K}=\{K(1),\ldots,K(p)\}^*\in\prod_{i=1}^{p}M_n,\quad \mathbf{K}_0=\{K_0(1),\ldots,K_0(p)\}^*\in\prod_{i=1}^{p}M_n$$

Here M_n is the Hilbert space of $(n\times n)$-matrices with the scalar product as defined above. The scalar product on a product of Hilbert spaces is defined in the usual manner, for example,

$$\langle\mathbf{K}_1,\mathbf{K}_2\rangle=\sum_{i=1}^{p}\langle K_1(i),K_2(i)\rangle,\quad \mathbf{K}_1,\mathbf{K}_2\in\prod_{i=1}^{p}M_n$$

The resulting Hilbert space is denoted by $\mathbf{H}$.

2. Construction of the data set.

Before the construction of the data set we present the following definitions. Let $x\in R^n$, $y\in R^k$.

Definition 2.1. The linear operator $x\otimes y$ from R^n into R^k prescribed by the formula

$$(x\otimes y)\,z=(x,\ z)\,y,\quad z\in R^n \tag{2.1}$$

is called the tensor product of vectors x and y.

The operator $x\otimes y$ can be written in matrix form as yx^*. Further, let A be a linear operator from R^n into R^m, B be a linear operator from R^k into R^l, $R^n\otimes R^k$ and $R^m\otimes R^l$ be the tensor products of the corresponding Hilbert spaces /8—14/.

Definition 2.2. The linear operator $A\otimes B$ from $R^n\otimes R^k$ into $R^m\otimes R^l$, prescribed an operators of form $x\otimes y$, $x\in R^n$, $y\in R^k$ by the formula

$$(A\otimes B)(x\otimes y)=Ax\otimes By \tag{2.2}$$

and continued by linearity onto the whole space $R^n\otimes R^k$, is called the tensor product of the linear operators A and B.

We remark that the tensor product of vectors from Definition 2.1 is not the tensor product of the corresponding one-column matrices from Definition 2.2. The space $R^n\otimes R^k$ and the space of linear operators from R^n into R^k, as well as the space $M_n\otimes M_n$ and the space of linear operators from M_n into M_n, are isomorphic. Linear operators on spaces R^n and $_n$ admit of the representation $\Sigma x_i\otimes y_i$ and $\Sigma A_i\otimes B_i$, respectively. The matrix notation of operator $A\otimes\mathbf{B}$ where $A\in M_p$ and $\mathbf{B}$ is a linear operator from M_n into M_n, has the form

$$A\otimes\mathbf{B}=\begin{Vmatrix}a_{11}\mathbf{B} & \ldots & a_{1p}\mathbf{B}\\ \vdots & & \vdots\\ a_{p1}\mathbf{B} & \ldots & a_{pp}\mathbf{B}\end{Vmatrix} \tag{2.3}$$

and, in particular, we can examine the operator $A\otimes\mathbf{B}$ on space $\mathbf{H}$. The corresponding mapping from $R^p\otimes M_n$ onto $\mathbf{H}$ is constructed by the rule

$$x\otimes X\to\{x_1X,\ \ldots,\ x_p\,X\},\quad x\in R^p,\ X\in M_n$$

with a subsequent continuation by linearity onto the whole space $R^p\otimes M_n$. It remains to observe that the following relations are valid:

$$Ax\otimes By=B\,(x\otimes y)\,A^*,\ \langle A,\ x\otimes y\rangle=(Ax,\ y) \tag{2.4}$$

$$(A\otimes B)^*=A^*\otimes B^*,\ (A\otimes B)\,K=BKA^*$$

Here x, y are vectors, A, B, K are matrices such that the corresponding expressions in (2.4) are meaningful.

We pass on to the construction of the data set. From (2.1)—(2.4) we can obtain that the linear mapping specified by conditions (1.1), (1.2) is

$$T\mathbf{K} + \xi = \mathbf{z};\ \mathbf{z} = \mathbf{y} - (E_p \otimes GB)\,\mathbf{u} \tag{2.5}$$

$$T = (E_p \otimes G)\left\{T_{x(0)} + \sum_{i=0}^{p-2} T_{u(i)}\mathbf{B}S_{i+1}\right\}, \quad \mathbf{B} = E_p \otimes (B^* \otimes E_n) \tag{2.6}$$

$$T_{x(0)} = E_p \otimes (x^*(0) \otimes E_n), \quad T_{u(i)} = E_p \otimes (u^*(i) \otimes E_n)$$

$$S_i\,\{K_1, \ldots, K_p\} = \{0_n, \ldots, 0_n, K_1, \ldots, K_{p-i}\},\ \{K_1, \ldots, K_p\} \in H$$

Here T is a linear operator onto space $\mathbf{H}$, S_i is the operator of right shift onto $\mathbf{H}$, 0_n is the null element of space M_n. In this case constraint (1.3) can be written as follows:

$$\langle \mathbf{K} - \mathbf{K}_0, \mathbf{N}(\mathbf{K} - \mathbf{K}_0)\rangle + (\xi, \xi) \leqslant \mu^2, \ \mathbf{N} = E_p \otimes N_*$$

where $\mathbf{N}$ is a linear operator onto space $\mathbf{H}$. By calculations typical for linear-quadratic problems we can establish the validity of the following statement.

Theorem 2.1. The data set $\mathbf{K}(p)$ is an ellipsoid in space $\mathbf{H}$, defined by the equality

$$\mathbf{K}(p) = \{\mathbf{K} \in \mathbf{H} \mid \langle \mathbf{K} - \mathbf{K}_*, \mathbf{M}(\mathbf{K} - \mathbf{K}_*)\rangle \leqslant \mu^2 - \varkappa^2$$

Here $\mathbf{M}$ is a block $(p \times p)$-matrix with elements

$$A_{ii} = N_* + x(0)\,x^*(0) + \sum_{k=0}^{p-1-i} Bu(k)\,u^*(k)\,B^* \otimes G^*G,\ i = 1, \ldots, p$$

$$A_{ij} = \left(Bu(j-i-1)\,x^*(0) + \sum_{k=j-i}^{p-1-i} Bu(k)u^*(k-j-i)\,B^*\right) \otimes G^*G, \quad i < j = 1, \ldots, p$$

$$A_{ij} = A_{ji},\ j < i = 1, \ldots, p$$

The center $\mathbf{K}_*$ of the data set $\mathbf{K}(p)$ is the solution of the matrix equation

$$\mathbf{MK} = \mathbf{K}(y) \tag{2.7}$$

whose right-hand side is defined by the relation

$$\mathbf{K}(y) = \mathbf{NK}_0 + \{G^*z(1)\,x^*(0), \ldots, G^*z(p)\,x^*(0)\}^* +$$
$$\left\{G^*\left(\sum_{k=2}^{p} z(k)\,u^*(k-2)\right)B^*,\ G^*\left(\sum_{k=3}^{p} z(k)\,u^*(k-3)\right)B^*, \ldots,\ G^*z(p)\,u^*(0)\,B^*,\ O_n\right\}^*$$

where $z(1), \ldots, z(p)$ are the coordinates of the block p-vector $\mathbf{z}$. The number $\varkappa^2$ is determined by the formula

$$\varkappa^2 = (\mathbf{z}, \mathbf{z}) + \langle \mathbf{K}_0, \mathbf{NK}_0\rangle - \langle \mathbf{K}_*, \mathbf{K}(y)\rangle \tag{2.8}$$

where vector $\mathbf{z}$ is constructed from the signal by virtue of the second relation in (2.5).

The solution of Eq.(2.7) for the center $\mathbf{K}_*$, as well as the determination of number $\varkappa^2$ from (2.8), can be reduced to a simpler problem since the operator $\mathbf{M}^{-1}$ admits of a convenient factorization which we now proceed to do.

3. Factorization of operator $\mathbf{M}^{-1}$ defining the center of data set $\mathbf{K}(p)$.

The following assertion can be verified directly.

Lemma 3.1. Let X and Y be finite-dimensional vector spaces, E_X and E_Y be identity operators on X and Y, respectively, T_1 be a linear operator from X into Y. Then the equality

$$(T_1^*T_1 + E_X)^{-1} = E_X - T_1^*(T_1T_1^* + E_Y)^{-1}T_1 \tag{3.1}$$

is valid.

If in Lemma 3.1 we set

$$X = \mathbf{H},\ Y = \prod_{i=1}^{p} R^n,\ T_1 = T\mathbf{N}^{-1/2}$$

then the relation

$$\mathbf{K}_* = \mathbf{K}_0 + \mathbf{N}^{-1}T^*(T\mathbf{N}^{-1}T^* + E_Y)^{-1}\{\mathbf{z} - T\mathbf{K}_0\} \tag{3.2}$$

follows from (2.7). Using definition (2.6), we obtain

$$T\mathbf{N}^{-1}T^* = (E_p \otimes G)\left\{T_{x(0)} + \sum_{i=0}^{p-2} T_{u(i)}\mathbf{B}S_{i+1}\right\}\mathbf{N}^{-1}\left\{T^*_{x(0)} + \sum_{i=0}^{p-2} S^*_{i+1}\mathbf{B}^*T^*_{u(i)}\right\}(E_p \otimes G^*) =$$

$$(E_p \otimes G)\left\{T_{x(0)}\mathbf{N}^{-1}T^*_{x(0)} + \sum_{i=0}^{p-2} T_{u(i)}\mathbf{B}S_{i+1}\mathbf{N}^{-1}T^*_{x(0)} + \sum_{i=0}^{p-2} T_{x(0)}\mathbf{N}^{-1}S^*_{i+1}\mathbf{B}^*T^*_{u(i)} + \sum_{i=0}^{p-2}\sum_{j=0}^{p-2} T_{u(i)}\mathbf{B}S_{i+1}\mathbf{N}^{-1}S^*_{j+1}\mathbf{B}^*T^*_{u(j)}\right\}(E_p \otimes G^*)$$

Henceforth, for simplicity we reckon that operator N_* has the form $N_* = N_1 \otimes N_2$, where N_1, N_2 are symmetric positive-definite matrices from space M_n. It can be verified that

$$T_{x(0)}\mathbf{N}^{-1}T^*_{x(0)} = E_p \otimes (x^*(0)N_1^{-1}x(0) \otimes N_2^{-1}) \tag{3.3}$$

Carrying out analogous computations for summands of the form

$$T_{u(i)}\mathbf{B}S_{i+1}\mathbf{N}^{-1}T^*_{x(0)},\ T_{x(0)}\mathbf{N}^{-1}S^*_{i+1}\mathbf{B}^*T^*_{u(i)},\ T_{u(i)}\mathbf{B}S_{i+1}\mathbf{N}^{-1}S^*_{j+1}\mathbf{B}^*T^*_{u(j)}$$

and taking into account (3.2), we can verify the validity of the next statement.

Theorem 3.1. If operator $N_* = N_1 \otimes N_2$, where N_1, N_2 are symmetric positive-definite matrices, then the center $\mathbf{K}_* = \{K_*(1), \dots, K_*(p)\}^*$ of data set $\mathbf{K}(p)$ is determined from the condition

$$K_*(1) = K_0(1) + N_*^{-1}G^*\left\{x_0 \otimes l_{*1} + \sum_{k=2}^{p}(u_{k-2} \otimes l_{*k})B^*\right\} \tag{3.4}$$

$$\dots\dots\dots\dots\dots\dots$$

$$K_*(p-1) = K_0(p-1) + N_*^{-1}G^*\{x_0 \otimes l_{*p-1} + (u_0 \otimes l_{*p})B^*\}$$

$$K_*(p) = K_0(p) + N_*^{-1}G^*\{x_0 \otimes l_{*p}\}$$

where $\mathbf{l}_* = \{l_{*1}, \dots, l_{*p}\}$ is the solution of the system

$$A(E_p \otimes GN_2^{-1}G^*)\mathbf{l}_* = \mathbf{z}_* \tag{3.5}$$

The elements a_{ij} of the symmetric positive-definite matrix $A \in M_p$ have the form

$$a_{11} = (x(0),\ N_1^{-1}x(0)), \quad a_{ii} = (x(0),\ N_1^{-1}x(0)) + \sum_{k=0}^{i-2}(u(k),\ B^*N_1^{-1}Bu(k)), \quad i = 2, \dots, p$$

$$a_{1i} = (x(0),\ N_1^{-1}Bu(i-2)), \quad a_{i1} = a_{1i},\ i = 2, \dots, p$$

$$a_{ij} = (x(0),\ N_1^{-1}Bu(j-i-1)) + \sum_{k=0}^{i-2}(u(k),\ B^*N_1^{-1}Bu(k+j-i)), \qquad a_{ji} = a_{ij},\ 2 \leqslant i < j \leqslant p$$

The block p-vector $\mathbf{z}_*$ has the form

$$\mathbf{z}_* = \mathbf{z} - (E_p \otimes G)\left\{K_0(1)x(0),\ K_0(2)x(0) + K_0(1)Bu(0),\ \dots,\ K_0(p)x(0) + \sum_{k=0}^{p-2}K_0(k-1)Bu(k)\right\}^*$$

To prove Theorem 3.1 it remains to note that

$$\mathbf{z} - \mathbf{z}_* = T\mathbf{K}_0,\ T^*\mathbf{l}_* = \left\{G^*l_{*1}x^*(0) + \sum_{k=2}^{p}G^*l_{*k}u^*(k-2)B^*,\ \dots,\ G^*l_{*p-1}x^*(0) + G^*l_{*p}u^*(0)B^*,\ G^*l_{*p}x^*(0)\right\}$$

Theorem 3.1 can be looked upon as an assertion on the factorization of operator $\mathbf{M}^{-1}$ in the form $\mathbf{R}\mathbf{Q}^{-1}$, where $\mathbf{R}(\mathbf{l}_*)$ is a computation operator (i.e., not requiring the solving of any equation if we take the matrices N_1^{-1}, N_2^{-1} as being defined beforehand) specified by formulas (3.4), while the operator $\mathbf{Q} = A(E_p \otimes GN_2^{-1}G_p^*)$ specified Eq. (3.5) on the succession space

$$\prod_{i=1}^{p} R^n$$

Thereby Theorem 3.1 enables us to avoid solving matrix equations even though the original extremal problem was prescribed *a priori* on the matrix space $\mathbf{H}$. In addition, the number $\varkappa^2$ defined in Theorem 2.1 by formula (2.8) can be computed only with respect to $\mathbf{l}_*$ — the solution of Eq. (3.5) — i.e., and here we need not go from the space

$$\prod_{i=1}^{p} R^n$$

to the matrix space **H**. An example of such a computation of number $\varkappa^2$ is considered below for a one-step model of the present problem.

To compare the equations for the center in Theorems 2.1 and 3.1 we turn to a one-step identification model

$$x(1) = Ax(0) + Bu(0) \tag{3.6}$$

under constraints (3.7) and measurable signal (3.8)

$$\langle A - A_0, \mathbf{N}(A - A_0)\rangle + (\xi, \xi) \leqslant \mu^2 \tag{3.7}$$

$$y = Gx(1) + \xi \tag{3.8}$$

We set $K(1) = A$, $z = y - GBu(0)$, $p = 1$ and we make use of the results in Sects.2 and 3. For simplicity we assume as well that $\mathbf{N}$ is the operator of left multiplication be a symmetric positive-definite matrix $N \in M_n$. In this case Theorem 2.1 leads to the following statement.

Corollary 3.1. When $p = 1$ the data ellipsoid **K** for system (3.6)—(3.8) is determined by the conditions

$$\mathbf{K} = \{A \in M_n \mid \langle A - A_1, \mathbf{M}(A - A_1)\rangle \leqslant \mu^2 - \varkappa^2\} \tag{3.9}$$

$$\mathbf{M} = E_n \otimes N + x(0)x^*(0) \otimes G^*G \tag{3.10}$$

A_1 is the solution of the matrix equation

$$NA + G^*GAx(0)x^*(0) = NA_0 + G^*zx^*(0), \quad z = y - GBu(0) \tag{3.11}$$

$$\varkappa^2 = (z, z) + \langle A_0, NA_0\rangle - \langle A_1, NA_0 + G^*zx^*(0)\rangle \tag{3.12}$$

Using arguments similar to those in (/13/, Chapter 8), we can write the solution of Eq. (3.10) as follows.

Corollary 3.2. If matrix G is of full rank and the commutator /12/ $[G^*G, N] = O_n$, then the solution of Eq.(3.10)— the center of elliposid **K** — has the form

$$A_1 = (G^*G)^{-1}\int_0^\infty \exp(-(G^*G)^{-1}Nt)\{NA_0 + G^*[y - GBu(0)]x^*(0)\} \times \exp(-x(0)x^*(0)t)\,dt \tag{3.13}$$

However, if the center A_1 of elliposid **K** is found by Theorem 3.1, then the resultant factorization leads to the following statement.

Corollary 3.3. When $p = 1$ the center of ellipsoid **K** for system (3.6)—(3.8) can be determined from the formula

$$A_1 = A_0 + N^{-1}G^*(x(0) \otimes l_*) \tag{3.14}$$

where l_* is the solution of the equation

$$l + \|x(0)\|^2 GN^{-1}G^*l = y - G\{A_0x(0) + Bu(0)\} \tag{3.15}$$

In this case

$$\varkappa^2 = (l_*, z - GA_0x(0)) = \|l_*\|^2 + \|x(0)\|^2 (l_*, GN^{-1}G^*l_*) \tag{3.16}$$

Representation (3.16) is obtained from (3.12), (3.14), (3.15) by direct verification. An analogous representation holds for the multistep model (1.1)—(1.3).

The operator T from (2.6) admits of a simpler representation, but for solving the continuous analog of the identification problem given by a limit transition, as well as for comparing the results in the discrete— and continuous— time representations, the expansion (2.6) of the present paper is more convenient. The solution of one continuous-time identification problem can be found in /7/.

The author thanks A.B. Kurzhanskii for attention to the work and for discussions.

REFERENCES

1. KRASOVSKII N.N., Theory of Control of Motion. Linear Systems. Moscow, NAUKA, 1968.
2. KURZHANSKII A.B., Control and Observation Under Conditions of Uncertainty. Moscow, NAUKA, 1977.
3. KURZHANSKII A.B., On adaptive control in mechanical systems. Teor. i Prikl. Mekhan. (Bolgaria), No.2, 1978.
4. KURZHANSKII A.B., GUSEV M.I., Multicriterial game-theoretic problems of control for systems with incomplete information. — In: Proc. 7th Congress Int. Federat. Automat. Control (IFAC). v. 2 Helsinki, p.1041, 1978.
5. KATS I.Ia. and KURZHANSKII A.B., Minimax multistep filtering in statistically uncertain situations. Avtom. i Telemekh., No.11, 1978.
6. KOSHCHEEV A.S., On state estimation for controlled systems under uncertain conditions. Differents. Uravnen., Vol.13, No.12, 1977.
7. ISAKOV A.I., On the estimation of the weight functions of linear controlled systems from measurement results. In: Optimal Control in Dynamic Systems. Sverdlovsk, Ural'sk. Nauch. Tsentr, Akad. Nauk SSSR, 1979.
8. SCHATTEN R., A Theory of Cross-Spaces, Annals of Math. Studies, No.26, Princeton, NJ. Princeton Univ. Press, 1950.
9. SCHATTEN R., Norm Ideals of Completely Continuous Operators. Berlin—Göttingen—Heidelberg, Springer-Verlag, 1960.
10. BOURBAKI N., Éléments de Mathématique, Livre II: Algèbre. Chapitre 1: Structures Algèbriques; Chapitre 2: Algèbre Linéaire; Chapitre 3: Algèbre Multilinéaire. Paris, Hermann et Cie., 1970. (English translation: Reading, MA, Addison-Wesley Publ. Co., 1974).
11. SCHAEFER H.H., Topological Vector Spaces. New York, The Macmillan Co., 1966.
12. GLAZMAN I.M. and LIUBICH Iu.I., Finite-Dimensional Linear Analysis in Problems. Moscow, NAUKA, 1969.
13. LANCASTER P., Theory of Matrices. New York—London, Academic Press, Inc., 1969.
14. HALMOS P.R., Finite-Dimensional Vector Spaces, Annals of Math. Studies, No.7, Princeton, NJ., Princeton Univ. Press, 1942.

Translated by N.H.C.